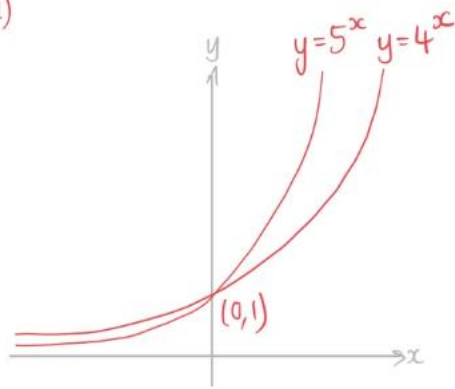


Q1a

a)



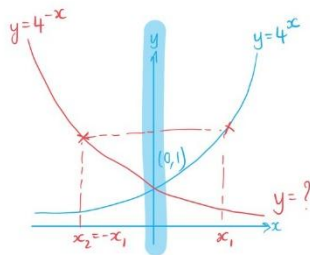
$y = a^x$
as $x \uparrow$ by 1 $y_1 \times 4$
 $y_2 \times 5$
 y_2 increases at faster rate $\therefore y_2$ has steeper gradient

asymptote: $y=0$ for both curves.

Q1b

(a) On the same axes, sketch the graphs of $y = 4^x$ and $y = 5^x$. Label any points of intersection with the coordinate axes. Write down the equations of any asymptotes.

(b) Write down an equation for the graph that is a reflection of $y = 4^x$ in the y-axis.



[4]

[1]

b) multiplying x by -1

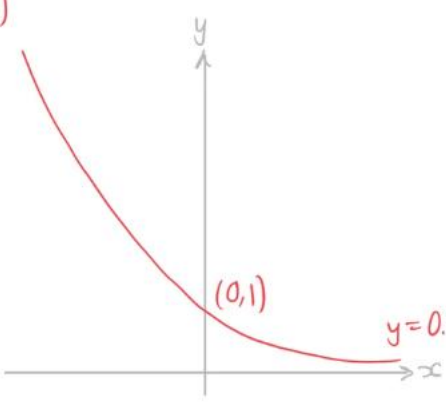
sub $-x$ into x

$$y = 4^{-x}$$

save my exams

Q2a

a) i)



$y = a^x$
 $a < 1$
 as $x \uparrow$ $y \downarrow$

$y = 0.4^x$ @ $x = 0$ $y = 0.4^0 = 1$

asymptote: $y = 0$

ii) exponential decay

(as x increases, y decreases exponentially)

Q2b

b)

$$0.064 = 0.4^x$$

$$0.4^3 = 0.064$$

$$4^3 = 64$$

$$x = 3$$

OR alternatively use logs!

$$x = \log_{0.4} 0.064 = 3$$

$$x = 3$$

Q3a

$$10^2 = 1000 \quad 3$$

$$10^3 = 10000 \quad 4$$

$$3 + 4 = 7$$

Q3b

$$6^2 = 36$$

$$\therefore \log_6 a = 2$$

what power do we raise 6 to, to get a? 2

$$6^2 = a = \boxed{36}$$

Q3c

$$\frac{2(3) + 3^3 - 1}{2} = \frac{6 + 27 - 1}{2} = \frac{32}{2} = \boxed{16}$$

Q4a

$$2(3) = x\left(\frac{1}{2}\right)$$

$$6 = \frac{x}{2}$$

$$\boxed{x = 12}$$

Q4b

$$3\log_4 x - \log_4 x = 3\log_5 25$$

$$2\log_4 x = 3(2)$$

$$\log_4 x = 3$$

$$4^3 = x$$

$$\boxed{64 = x}$$

Q5

$$2^{2x} = (2^x)^2$$

① Spot the hidden quadratic

$$2(2^x)^2 + 4 = 9(2^x)$$

② Substitute $y = 2^x$

$$2y^2 + 4 = 9y$$

$$2y^2 - 9y + 4 = 0$$

$$2y^2 - y - 8y + 4 = 0 \quad \begin{array}{l} \times 8 \\ + -9 \\ -1, -8 \end{array}$$

$$y(2y-1) - 4(2y-1) = 0$$

$$(y-4)(2y-1) = 0$$

$$y = 4, \frac{1}{2}$$

$$2^x = 4, \frac{1}{2}$$

③ Solve

(i) observation

$$2^2 = 4$$

$$2^{-1} = \frac{1}{2}$$

$$x = 2, -1$$

or (ii) logs!

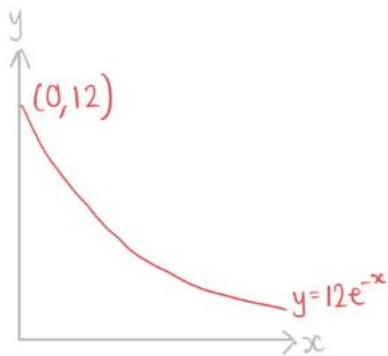
$$a = b^x$$

$$x = \log_b a$$

$$x = \log_2 4, \log_2 \frac{1}{2}$$

$$x = 2, -1$$

Q6a

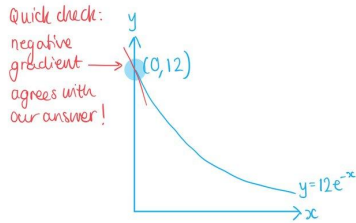


asymptote: $y = 0$

Q6b

(a) Sketch the graph of $y = 12e^{-x}$ for $x \geq 0$.
Label any points of intersection with the coordinate axes.
Write down the equations of any asymptotes.

(b) Write down the gradient of $y = 12e^{-x}$ at the point where $x = 0$.



[3]

$$\frac{dy}{dx} = 12(-1)e^{-x} = -12e^{-x}$$

sub in $x=0$

[1]

$$\frac{dy}{dx} = -12e^{-0} = \boxed{-12}$$

Q7a

Sub $2x$ for x

$$f(2x) = 3e^{2(2x)} = 3e^{4x}$$

$$\boxed{f(2x) = 3e^{4x}}$$

Q7b

① differentiate

$$f'(x) = 3(2)e^{2x}$$

② sub $2x$ into x

$$f'(2x) = 6e^{2(2x)} = 6e^{4x}$$

$$\boxed{f'(2x) = 6e^{4x}}$$

Q8

Spot the hidden quadratic in e^x !

$$2(e^x)^2 - (e^x) - 10 = 0$$

let $y = e^x$

$$2y^2 - y - 10 = 0$$

$$(2y-5)(y+2) = 0$$

$$y = \frac{5}{2}, -2$$

$$e^x = \frac{5}{2} \quad \cancel{e^x = -2}$$

$$\ln \frac{5}{2} = \boxed{x = 0.916} \quad (3sf)$$

Q9a

$$\frac{dy}{dx} = a(b)e^{bx} = \boxed{abe^{bx}}$$

Q9b

Sub $x=0$ into eqn (a)

$$\frac{dy}{dx} = a b e^{b(0)^1} = ab = 12$$

$$\boxed{b = \frac{12}{a}}$$

Q9c

$y(a, b, x)$ →
sub $b = \frac{12}{a}$ into
eqn in (a)
←
 $y(a, x)$

$$y = ae^{\frac{12}{a}x}$$

Q10a

$$e^{-x} = \frac{1}{e^x}$$

$$e^x - \frac{1}{e^x} = 0$$

$$(e^x)^2 - 1 = 0 \quad \text{difference of 2 squares}$$

$$(e^x + 1)(e^x - 1) = 0$$

$$\boxed{e^x = 1} \quad \boxed{\cancel{e^x = -1}}$$

← $e^x > 0 \therefore$ this eqn has
no real solutions

$$\downarrow$$
$$x = 0$$

so $x = 0$ is the only real solution

Q10B

$$\text{sub } e^{-x} = \frac{1}{e^x}$$

$$e^x + \frac{1}{e^x} = 0$$

$$(e^x)^2 + 1 = 0$$

$$(e^x)^2 = -1$$

$$e^x = \sqrt{-1} \leftarrow \text{has no real solutions.}$$

$$\text{so } e^x + e^{-x} = 0 \text{ has no real solutions.}$$